HYDROSTATIC TESTS OF STRUCTURAL MODELS FOR PRELIMINARY DESIGN OF A WEB-STIFFENED SANDWICH PRESSURE HULL

by

Kenneth Hon and William F. Blumenberg

STRUCTURAL MECHANICS LABORATORY
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A series of structural model tests was conducted to evaluate a preliminary sandwich design of the pressure hull for a proposed 15,000-ft operating-depth oceanographic vehicle. The model studies were limited to an evaluation of the static strength of a typical section of the web-stiffened sandwich configuration. It is intended that the prototype be fabricated of high-strength, $\alpha$-titanium alloy having a nominal yield strength of approximately 120,000 psi.

Model tests in the series indicated that the original design would not prove adequate to meet the minimum specified collapse pressure of 10,000 psi; this is attributed primarily to a premature inelastic general-instability mode of failure. A redesign has been suggested with the objective of increasing the overall stability of the hull without increasing the weight.

A small-scale model, machined from a nascent bar of titanium alloy, was also tested to determine whether the suggested redesign represents a significant improvement in static strength over the original design. Upon extrapolation of the observed collapse pressure for the redesigned model to a yield strength of 120,000 psi, a pressure of 11,290 psi is obtained. Here again, failure occurred by general instability at a reduced modulus but at a much higher pressure than for the original design. The results indicate that the static strength depends on the stress-strain characteristics of the material in the hull structure. Built-in stresses, such as those that might be induced by rolling and welding of flat plates into the cylindrical form, could significantly alter the shape of the stress-strain curve so that the collapse strength of a fabricated structure may be considerably lower than that realized from tests with perfectly circular and initially stress-free machined models.

Before any final conclusions can be drawn concerning the adequacy of the suggested redesign, it will be necessary to test larger scale titanium models which are fabricated to be identical in all respects, except size, to those anticipated in the prototype. In this way all factors which appear to influence static and fatigue strength will be adequately considered.
INTRODUCTION

A preliminary hull design for an oceanographic vehicle intended for operation at 15,000 ft was submitted\(^1\) to the David Taylor Model Basin by the Bureau of Ships for evaluation and recommendation for possible redesign, if this proved necessary. A web-stiffened sandwich cylinder with a weight-displacement ratio of approximately 63 percent, to be fabricated of α-titanium having a yield strength of 120,000 psi, constitutes the basic hull design. The minimum specified design collapse pressure is 10,000 psi. A schematic of the proposed design is shown in Figure 1.

The Bureau of Ships preliminary design was arrived at by using an engineering-type analysis based primarily on previous test results of web-stiffened sandwich cylinders;\(^2\) however, these cylinders were intended for a collapse depth of about 6000 ft. A description of this procedure is outlined in the next section of this report.

To evaluate the static strength of the proposed hull design, five structural models were designed and tested to collapse under external hydrostatic pressure. Two of these models were one-diameter-long cylinders fabricated from HY-100 steel plating which was available at the time. Three other models were machined from bar stock of titanium alloy; one model was one diameter long and two models were four diameters long.

In this report, construction details of the models are presented, the test procedures used are described, the experimental and the theoretical strength data are compared, and certain extrapolations are ventured as to the structural behavior of the prototype hull.

DESIGN AND DESCRIPTION OF MODELS

Three different designs of a web-stiffened, sandwich-type pressure hull were investigated. The titanium hulls were designed to have the same weight, the same outside diameter, and the same design collapse strength (10,000 psi) for the fabricated prototype structure. Figure 2 shows the

\(^1\) References are listed on page 35.
full-scale dimensions of the three designs and the corresponding models representative of each design.

**MODEL OV-1**

This model represents a one-diameter length of the hull design as proposed by the Bureau of Ships, see Figures 1 and 2a. The design was arrived at by using a modified membrane-type analysis in lieu of a more exact method of determining a suitable geometry and material distribution for the sandwich structure. This membrane-type analysis was used to determine the shell thickness at which hoop yielding would occur at a
Figure 2 - Sandwich Cross Sections Showing Prototype Dimensions of Models Tested

W/D is the ratio of the weight of the titanium (0.16 lb/in.\(^3\)) hull over the weight of sea water (64 lb/ft\(^3\)) displaced by the hull.
pressure of 10,000 psi, according to the simple relation \( \sigma_y = \frac{pR}{h} \), in an unstiffened cylinder of the same outside diameter as the prototype. The shell thickness was then calculated in a like manner for an unstiffened cylinder of the same diameter as a "model" of structural design similar to the prototype and subjected to a pressure equal to the collapse pressure realized from tests of that "model." The "model" chosen as a datum was Model SP-3, a web-stiffened sandwich cylinder, the results of which are presented in Reference 2. With this latter value of shell thickness, the weight per unit of displaced volume was determined for the unstiffened cylinder. Then the shell thickness calculated for the unstiffened cylinder, based on the proposed prototype requirements, was reduced by the ratio of the unit weight of Model SP-3 to that of the equivalent unstiffened cylinder. This reduced thickness for the proposed prototype was then distributed in the form of a web-stiffened sandwich cross section having the same weight per unit length. For convenience, the two coaxial cylindrical shells and the web stiffeners were specified as having the same thickness; however, subsequent calculations have shown that this need not represent optimum distribution of the material.

The first model tested (Model OV-1) was fabricated of high strength steel and was intended primarily to determine the interbay strength of the typical sandwich section and to verify the axisymmetric elastic behavior predicted by the analysis of Reference 3. Also, the long lead time and great cost required to obtain the necessary titanium material and the lack of well-defined fabrication procedures to build a welded titanium model within a short time necessitated the use of an equivalent-strength steel to check the basic design.

The dimensions and design details of Model OV-1 are shown in Figure 3. These dimensions were dictated by the size of high-pressure test facilities then existing at the Model Basin and by the availability of suitable thickness material on hand. The entire model was fabricated from a single steel plate having an average thickness of 0.242 in. and heat-treated to a compressive yield strength of approximately 115,000 psi. The yield strength of the shell material ranged from a low of about 109,000 to a high of about 117,000 psi. Yield strength values of 114,000 and 111,000
psi were estimated for the original flat plating and as close as possible to the regions of failure observed on the inner and outer shells of the model, respectively.

The webs of the sandwich section were made of three equal segments cut from the flat plating and butt-welded together to form complete circular rings. After the inner shell was rolled and welded, each web was positioned and welded continuously to the inner shell. The outer shell was slotted, rolled into a cylindrical form, and placed outside the web-and-inner shell assembly. The outer shell was then plug-welded to the webs at those points where the slots were cut.
The model was designed using an end arrangement to preclude premature failure near the rigid closure bulkheads; see Figure 3. It was made pressure-tight by welding heavy rings on each end and then attaching a heavy closure bulkhead to each ring.

MODELS OV-3 AND OV-4

The next step in the sequence of developing background information before going to some large-scale fabricated titanium models was to test some small-scale machined titanium models which could be made immediately at a very nominal cost. For this reason, Models OV-3 and OV-4, representing one-diameter and four-diameter lengths, respectively, of the original design, were built and tested.

The dimensions and design details of Models OV-3 and OV-4 are shown in Figure 4. Dimensional tolerances between the outer shell and the webs were specified so that a maximum clearance of 0.0008 in. between these two elements would exist when the outer shell was slipped over the inner shell-web combination. Figure 5 shows the component parts of Model OV-4, prior to assembly.

After each model was assembled, the ends of the cylinders were ground flush to ensure proper distribution of the axial pressure load to the inner and outer shells. The models were designed using an end arrangement, as shown in Figure 4, to preclude premature failure near the rigid closure rings. They were made pressure-tight by using a heavy plate and an "O"-ring seal at each end. The closure plates were not physically attached to the cylinder but were seated in place by the pressure. Figure 6 is an assembly drawing of Model OV-3.

Models OV-3 and OV-4 were machined from a 2 5/8-in.-diameter bar of 6Al-4V α-titanium alloy. Sixteen compression specimens were taken from the bar; the yield strength, based on a 0.2-percent offset, ranged from a low of 126,000 psi to a high of 145,000 psi with an average value of about 138,000 psi. Since the yield strength and stress-strain curve for titanium alloys are sensitive to the rate of loading, these specimens were tested at a rate corresponding to the pressure loading of the models. The rate of loading was equivalent to a stress-intensity level of 550 psi/min,
Figure 4 - Details of Models OV-3 and OV-4
Figure 5 - Model OV-4 Prior to Assembly

Figure 6 - Assembly Drawing of Model OV-3
computed on the basis of the Hencky-Von Mises criterion for a monolithic thick cylinder of equivalent weight.

The one-diameter-long model (OV-3) was tested to determine whether an increase in strength for a titanium hull similar to that observed for the steel model, OV-1, could be realized as a consequence of the effects of strain hardening. In addition, it was felt that this test of Model OV-3 would provide information on the feasibility of constructing models by slipping one shell over the other, and also would indicate whether this novel technique involved a loss of structural strength, if any existed, compared to the method utilized for Model OV-1, where the outer shell of the sandwich section was physically joined to the webs by slot welding.

The four-diameter-long model (OV-4) was intended to determine the collapse pressure and mode of failure for the overall cylindrical compartment of the original design. This model was considered necessary since the general-instability strength could not be determined by testing short-length cylinders such as Models OV-1 and OV-3.

**MODEL OV-2**

By the use of the elastic analysis of Reference 3, it was possible to predict high bending stresses in both shells between adjacent webs of the original sandwich design; in addition, the webs were found to be under-stressed. These results indicated that an improved design could be obtained, without increasing the weight of the structure, by a redistribution of the material. Several improved designs were analyzed. One of these designs is shown in Figure 2b and is represented by Model OV-2. Calculations indicated that the OV-2 configuration represented a more balanced design than that originally proposed. That is, the spread between the maximum and the minimum shell stresses is reduced in an attempt to more uniformly stress the entire sandwich cross section. It should be noted, however, that a "balanced stress design" need not represent the optimum structure for a given weight when the instability modes of failure are also of significance, as in the present design of a long hull compartment. Figure 7 presents a comparison of the computed stresses in the critical regions of a typical sandwich section for each of the three designs investigated.
Figure 7 - Theoretical Stress Intensity in Critical Shell Regions for the Three Designs Investigated

NOTE: Stress values were computed from analysis of Reference 3, with an assumed value of 0.33 as Poisson's ratio, and are given in psi per psi of pressure. Values shown in this figure represent the circumferential and longitudinal shell stresses and are written vertically and horizontally, respectively.

MODELS OV-1, OV-3, AND OV-4 (ORIGINAL DESIGN)

MODELS OV-2 (BALANCED STRESS DESIGN)

MODELS OV-5 (SUGGESTED REDESIGN)
The primary purpose of Model OV-2 was to determine whether an increase in interbay strength exists over that of the original design and to verify the elastic behavior predicted by the analysis of Reference 3. It was felt that these objectives could be accomplished by fabricating a model from steel plating of only about one diameter in length. Thus, Model OV-2 affords a direct strength comparison with Model OV-1.

The dimensions and design details of Model OV-2 are shown in Figure 8. Fabrication procedures for this model were identical to those for Model OV-1. The inner shell had an average thickness of 0.296 in.
and a yield strength ranging from 104,000 psi to 108,000 psi. The outer shell had an average thickness of 0.259 in. and a yield strength ranging from 109,000 psi to 113,000 psi. Yield strength values of 104,000 psi and 109,000 psi were estimated for the original flat plates and as close as possible to the regions of failure observed in the inner and outer shells of the model, respectively.

MODEL OV-5

From the tests of the titanium models, OV-3 and OV-4, it was conclusively determined that inelastic general instability is the critical mode of failure for the original design. Both of these models had the same typical interbay configuration. The only difference was in the overall length; OV-3 was one diameter long, whereas OV-4 was four diameters long. As a consequence of its shorter length, Model OV-3 had an elastic general-instability pressure twice that of the longer model. The observed collapse pressures of these two models reflected this difference. When extrapolated to the specified yield strength of 120,000 psi, the shorter model (OV-3) failed at a pressure 10 percent above the minimum specified collapse pressure and the longer model (OV-4) failed 3.5 percent below this value.

It appeared that a redesign of the basic sandwich configuration to increase the general-instability strength was necessary to ensure that the fabricated structure would meet the minimum design pressure. One redesign possibility would be to increase the depth of the webs at the expense of making them thinner so as to retain the same overall weight of hull structure. Another possible redesign would be to decrease the overall stress intensity through the sandwich cross section by adding additional material to the two cylindrical shells. Either of these two alternatives, or a combination of them, may be necessary to attain the stated objective.

A small-scale, four-diameter-long, machined model (OV-5) incorporating the first alternative was designed and constructed. The overall depth of the sandwich cross section in this design corresponds to 12.9 in. as compared with 10 in. in the original design (see Figure 2); the web thickness was reduced to $1\frac{3}{8}$ in. in order to retain, as closely as possible,
the original hull weight. The elastic general-instability strength for the four-diameter-long redesigned model (OV-5) is exactly the same as that for the one-diameter-long model (OV-3) based on the original design.

The dimensions and design details of Model OV-5 are shown in Figure 9. Construction of this model was similar to that of Models OV-3 and OV-4. It was machined from a $4\frac{1}{8}$-in.-diameter bar of 6Al-4V α-titanium alloy. Six compression specimens were taken from the bar; the yield
strength, based on a 0.2-percent offset, ranged from a low of 121,000 psi to a high of 132,300 psi with an average value of about 127,500 psi. The specimens and model were loaded at twice the rate used for Models OV-3 and OV-4.

INSTRUMENTATION AND TEST PROCEDURE

To study the elastic and inelastic behavior of the structure and to facilitate interpretation of the mode of failure and collapse pressure, strains were measured on Models OV-1 and OV-2 by electrical resistance strain gages. Strain-gage locations for Models OV-1 and OV-2 are shown in Figures 10 and 11, respectively, together with the measured strain sensitivities. Because of the small size of Models OV-3, OV-4, and OV-5, it was not feasible to take strain measurements on these models.

Figure 10 - Theoretical and Measured Strain Sensitivities for Model OV-1
Water was used as the pressurizing medium in the tests of Models OV-1 and OV-2. They were tested to failure in a number of pressure runs; the maximum pressures reached during each run were as follows:

<table>
<thead>
<tr>
<th>Run</th>
<th>Maximum Pressure, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model OV-1</td>
</tr>
<tr>
<td>1</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>1,000</td>
</tr>
<tr>
<td>3</td>
<td>5,400</td>
</tr>
<tr>
<td>4</td>
<td>8,500</td>
</tr>
<tr>
<td>5</td>
<td>10,300</td>
</tr>
<tr>
<td>6</td>
<td>11,700 (collapse)</td>
</tr>
</tbody>
</table>

Figure 11 - Theoretical and Measured Strain Sensitivities for Model OV-2
Models OV-3, OV-4, and OV-5 were tested under external hydrostatic pressure in relatively small-size pressure tanks so as to restrict the extent of failure and thus to better determine the mode and point of initiation of failure. A light-grade oil was used as the pressurizing medium. These models were each tested in two pressure runs. For the first run, pressure was increased in increments of 500 psi until a maximum pressure of 5000 psi was reached. Each model was removed from the tank at the conclusion of its first run and was visually inspected for any possible leakage of oil into the inside of the model. For the second run, each model was pressurized to 5000 psi and pressure was then increased in increments of 100 psi until collapse occurred. The pressure was maintained for 2 min at each pressure interval for Models OV-3 and OV-4 and for 1 min for Model OV-5.

TEST RESULTS AND DISCUSSION

MODELS OV-1 AND OV-2

The ultimate load-carrying capacity of both Models OV-1 and OV-2 was 11,700 psi. The failure of Model OV-1 was restricted to an axisymmetric crease extending almost completely around the entire circumference between Webs 10 to 12. Figure 12 shows the mode of collapse. The collapse mode for Model OV-2 was one of apparent general instability in which a longitudinal crease extended almost the entire length of the model; see Figure 13.

Strain-sensitivity factors for each gage determined by averaging the results from each of the pressure runs are shown in Figures 10 and 11 for Models OV-1 and OV-2, respectively. Also given are the theoretical strains computed using the elastic analysis of Reference 3 for a typical bay of the sandwich cross section. The measured strains in the center region of both models compared favorably with the corresponding theoretical value, except that there was an appreciable variation in the observed circumferential strains in the outer shell at the toe of the webs of Model OV-1. This probably can be attributed to local distortion due to the slot-welding.
Pressure-strain plots for gages located in the region of failure of Model OV-1 are presented in Figures 14 and 15. An examination of these plots shows that at the higher pressures approaching collapse the strains could conceivably have been on the order of 20,000 to 30,000 μin./in. and possibly higher. Similar observations were also noted during the test of Model OV-2.

Stress-sensitivity factors calculated from the analysis of Reference 3 are given in Figure 7, which shows that the most critically stressed regions are located on the inner surface of the outer shell at the toe of the webs. Based on the theoretical stresses at this location, yielding according to the Hencky-Von Mises criterion would initiate at pressures of 7500
Figure 14 - Strain Plots of Gages Located at Station 10 1/2 on Model OV-1

and 9750 psi (assuming a yield strength of 120,000 psi) for Models OV-1 and OV-2, respectively. Unfortunately, the stresses in these locations could not be measured experimentally so as to provide a check on the computed values. This verification, however, could be and was done for the accessible regions of the outer and inner shells. For comparison purposes, Table 1 gives the experimental and theoretical Hencky-Von Mises stress ratios between adjacent webs for both shells. Also given in that table are the stress ratios for three other web-stiffened sandwich models recently tested at the Model Basin. In general, the midbay stresses determined by the theory of Reference 3 are in good agreement with the observed stresses except for the outer shell of Model OV-1, where the prediction was 35 percent below measurement. It should be noted, however, that this value was based on strains measured at only one location,
Figure 15 - Strain Plots of Gages Located at Station 11 1/2 on Model OV-1

<table>
<thead>
<tr>
<th>TABLE 1</th>
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<tr>
<td>Ratio of Experimental to Theoretical Hencky-Von Mises Stresses at Midbay Location</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>$\left(\frac{\sigma_{HVM}}{\sigma_{HVM, Theor.}}\right)_{\text{Exper.}}$</th>
<th>$\left(\frac{\sigma_{HVM}}{\sigma_{HVM, Theor.}}\right)_{\text{Theor.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OV-1</td>
<td>1.35 (1)*</td>
<td>1.02 (2)</td>
</tr>
<tr>
<td>OV-2</td>
<td>0.96 (3)</td>
<td>1.12 (4)</td>
</tr>
<tr>
<td>SP-3</td>
<td>0.97 (2)</td>
<td>0.99 (3)</td>
</tr>
<tr>
<td>SP-4</td>
<td>1.13 (5)</td>
<td>1.06 (5)</td>
</tr>
<tr>
<td>SP-9</td>
<td>1.12 (9)</td>
<td>1.00 (9)</td>
</tr>
</tbody>
</table>

*Numbers in parentheses denote number of gaged locations used for determining average experimental value.
which might possibly have been influenced significantly by the slot welds in the adjacent areas.

MODELS OV-3, OV-4, AND OV-5

The experimental collapse pressures for each of the three machined titanium models were:

<table>
<thead>
<tr>
<th>Model</th>
<th>Collapse Pressure psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>OV-3</td>
<td>12,700</td>
</tr>
<tr>
<td>OV-4</td>
<td>11,100</td>
</tr>
<tr>
<td>OV-5</td>
<td>12,000</td>
</tr>
</tbody>
</table>

Figures 16, 17, and 18 are photographs showing the mode of collapse for each model. A catastrophic failure was observed for Model OV-3. It appears that axisymmetric yielding of the shells between the web stiffeners may have been a contributing factor to collapse. Some permanent deformation in the form of an axisymmetric corrugation between the webs on both the inner and outer shells was visible over the entire structure.

Model OV-4 failed by general instability in an oval \( n = 2 \) mode without any rupturing of the shell elements. Unlike Model OV-3, permanent deformations between adjacent webs were observed only in limited regions of the structure.

Model OV-5, like OV-4, failed by general instability; however, there was a marked difference in the appearance of the failure. In addition, permanent deformations between adjacent webs were visible over the entire structure.
Figure 16 - Model OV-3 after Collapse
Figure 17 - Model OV-4 after Collapse
Figure 18 - Model OV-5 after Collapse
INTERPRETATION OF RESULTS

AXISYMMETRIC MODE OF COLLAPSE

Table 2 gives "upper bound" theoretical collapse pressures $p_t$ for each model. These pressures are based on considerations of an equivalent unstiffened cylinder having the same weight-displacement ratio and material properties as the corresponding sandwich cylinder. Also, the Hencky-Von Mises yield criterion was applied to the three-dimensional state of stress at midplane, as determined from the Lamé solution for a thick-walled cylinder. The "upper bound" pressures computed in this manner represent a state of complete stressing of the entire cross section up to the value of yield strength used in each case. These calculations are predicated on a plateau-type stress-strain curve, and the possibility of an increase in the strength of the material due to strain-hardening is not considered.

It can be seen in Table 2 that the aforementioned theoretical collapse pressure $p_t$ is considerably lower than the observed collapse pressure for each of the two steel models. Strain measurements taken during the tests of OV-1 and OV-2 indicated that these structures were grossly strained prior to collapse.

**TABLE 2**

Collapse Pressures

<table>
<thead>
<tr>
<th>Model</th>
<th>Shell Yield Strength psi</th>
<th>Material</th>
<th>Approximate Length-Diameter Ratio</th>
<th>Experimental Collapse Pressure psi</th>
<th>$P_c$ Three-Dimensional State of Stress over Entire Material psi</th>
<th>Elastic General-Instability Pressure psi</th>
<th>Inelastic General-Instability Pressure psi</th>
<th>Experimental Collapse Pressure &quot;Corrected&quot; to Yield Strength of 120,000 psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>OV-1</td>
<td>112,500*</td>
<td>Steel</td>
<td>1</td>
<td>11,700</td>
<td>10,310</td>
<td>100,000</td>
<td>--</td>
<td>12,480</td>
</tr>
<tr>
<td>OV-2</td>
<td>106,300*</td>
<td>Steel</td>
<td>1</td>
<td>11,700</td>
<td>9,940</td>
<td>101,000</td>
<td>--</td>
<td>13,210</td>
</tr>
<tr>
<td>OV-3</td>
<td>138,000</td>
<td>Titanium</td>
<td>1</td>
<td>12,700</td>
<td>12,660</td>
<td>51,000</td>
<td>12,200</td>
<td>11,040</td>
</tr>
<tr>
<td>OV-4</td>
<td>138,000</td>
<td>Titanium</td>
<td>4</td>
<td>11,100</td>
<td>12,660</td>
<td>25,450</td>
<td>11,250</td>
<td>9,650</td>
</tr>
<tr>
<td>OV-5</td>
<td>127,500</td>
<td>Titanium</td>
<td>4</td>
<td>12,000</td>
<td>11,990</td>
<td>51,000</td>
<td>11,960</td>
<td>11,290</td>
</tr>
</tbody>
</table>

* Weighted value based on the relative circumferential membrane stresses in each of the outer and inner shells.
To determine how much straining is necessary before the material in the shells is in the strain-hardening range, a compression specimen was taken from a \( \frac{1}{4} \)-in.-thick steel plate which was heat-treated to a yield strength of approximately 100,000 psi. Figure 19 is the stress-strain curve for this specimen tested under uniaxial compressive loading. The specimen was loaded at a slow rate; that is, 1 hr was needed to reach the strain-hardening range. The observed stress-strain characteristics indicated that strain hardening for a material similar to that used in Models OV-1 and OV-2 would be reached at a strain of about 17,000 \( \mu \text{in./in.} \). If the material was worked into this range, it would, in effect, correspond to raising the yield strength. It is conceivable that the strains in Models OV-1 and OV-2, at pressures approaching collapse, were on the order of 20,000 to 30,000 \( \mu \text{in./in.} \), thus suggesting the possibility of strain hardening. Compression specimens taken from the shells of the models after collapse, and as close to the regions of failure as possible, gave average increases in yield strength of 15 and 19 percent over the values obtained.

![Figure 19 - Stress-Strain Curve of Specimen Taken from 1/4-Inch-Thick HY-80 Steel Plate](image-url)
from the original flat plating of Models OV-1 and OV-2, respectively. These higher yield strengths are reflected in the 13- and 18-percent differences between observed collapse pressure and the $p_r$ pressure for each of the models. Thus, the optimistic collapse strengths realized from the tests of both of these models can be attributed to the beneficial effects of strain hardening of the shell material.

When the observed pressures of Models OV-1 and OV-2 are extrapolated to a common yield strength of 120,000 psi, as shown in the last column of Table 2, OV-2 has a higher collapse pressure. This can be attributed to the more "balanced state of stress" for this configuration; see Figure 7.

Table 2 also lists the "upper bound" pressures $p_r$ for each of the machined titanium models. The almost exact agreement between $p_r$ and the observed collapse pressure in the case of Models OV-3 and OV-5 is fortuitous. However, it indicates the high degree of efficient utilization of the material that can be realized with sandwich construction. For Model OV-3, a comparison can be made with the counterpart steel sandwich model (OV-1) where an appreciable difference between observation and "upper bound" prediction can be attributed to the fact that the steel structure was stressed into the strain-hardening range. The effects of strain hardening were not realized with the titanium sandwich model (OV-3), even though the geometric disposition of the material was the same in both cases.

If a linear "correction" of the experimental collapse pressure for the short titanium model (OV-3) is made to allow for the higher yield strength of the shell material as compared to the minimum specified, a collapse pressure of 11,040 psi is realized. This value is indicative of the maximum collapse strength which can possibly be realized with the sandwich configuration shown in Figure 1, but it does not reflect the influence of residual stresses which may arise due to rolling and welding in a fabricated prototype. These considerations would have a tendency to "round off" the stress-strain curve even more so than that shown in Figure 20. Also, coupled with the destabilizing influence of a long compartment length, they could lead to a value of collapse strength below 11,040 psi. The question as to how much lower this strength would be and whether it would fall below the minimum design collapse pressure of 10,000 psi,
Figure 20 - Typical Stress-Strain Curve of Compression Specimen Taken from Nascent Titanium Alloy Bar Used in Models OV-3 and OV-4

has been partially answered by the tests of the four-diameter-long machined Models OV-4 and OV-5.

GENERAL-INSTABILITY MODE OF COLLAPSE

The tests of Model OV-4, which was four diameters long, were intended to provide an insight into the possible reduction in collapse strength and change in mode of failure which might be expected in the case of a long compartment. The tests of Model OV-5 were intended to determine whether a redesign, slanted toward increasing the general-instability
strength at the expense of higher stresses (see Figure 7), would give rise to higher overall collapse strength. Based on the theoretical stresses given in Figure 7 and using a yield strength of 120,000 psi, yielding according to the Hencky-Von Mises criterion would initiate at the most highly stressed region of Model OV-5 at a pressure of 7240 psi as compared with a pressure of 7500 for the configuration of Model OV-4.

Both models failed by inelastic general instability. Although no rigorous method presently exists for computing the elastic general-instability strength of sandwich cylinders, an extension of the well-known criteria and formulas used for conventional ring-stiffened cylinders permits an order-of-magnitude determination for the present cases. The well-known and convenient Bryant formula was used for this purpose. Calculation of the "shell-term" was based on the assumption that the two sandwich shell elements constituted the membrane contribution to the buckling strength; the combined shell thickness was assumed to be at a radius equidistant between the radii of the two concentric cylinders. The bending contribution to the buckling strength, that is, the "frame-term" in Bryant's formula, was computed for a "free" ring having the actual sandwich cross section shown in Figure 2 for each case.

Table 2 lists the elastic general-instability pressures computed by the above procedure, using elastic moduli of $16 \times 10^6$ psi for titanium and $30 \times 10^6$ psi for steel. It is seen that the elastic general-instability pressure for the four-diameter-long model of the original design (OV-4) is only one-half that for both the one-diameter-long model (OV-3) and the four-diameter-long model of the suggested redesign (OV-5).

The elastic general-instability pressures listed in Table 2 are based on the assumption that the stresses in the sandwich structure are within the proportional limit of the material. However, this was not the case for the models tested. For Models OV-4 and OV-5, the elastic-instability pressures were effectively reduced as a consequence of the curvilinear nature of the stress-strain curve (see Figure 20) for the titanium material to such a state that inelastic general instability, or buckling at a reduced modulus, became the determinative factor of collapse.

To determine the inelastic general-instability collapse pressure $p_c$ for cases where the stresses exceed the proportional limit, the following
formula was used:

\[ p_c = p_s \frac{\sqrt{E_s E_t}}{E} + p_f \frac{E_t}{E} \]  

where \( p_s \) and \( p_f \) are the shell and frame terms, respectively, in Bryant's formula, and \( E, E_s, \) and \( E_t \) are the elastic, secant, and tangent moduli, respectively, of the material. Equation [1] has found considerable experimental verification, as reported in Reference 6.

Above the proportional limit, the secant and tangent moduli vary with the stress intensity as seen in Figure 20. Therefore, Equation [1] cannot be used directly to determine the collapse pressure \( p_c \); a knowledge of the membrane stresses in the sandwich structure is required to first determine a stress intensity which is then used in conjunction with the uniaxial stress-strain curve to determine appropriate values of \( E_s \) and \( E_t \). The following membrane stresses were assumed for the sandwich cross section:

**Circumferential stress:**

\[ \sigma_\phi = \frac{2(R_o + \frac{1}{2} h_o) L_F}{2 A_T} \quad p \equiv K_1 p \]  

**Longitudinal stress:**

\[ \sigma_x = \frac{\pi (R_o + \frac{1}{2} h_o)^2}{2 \pi (R_o h_o + R_i h_i)} \quad p \equiv K_2 p \]

where \( h_i, h_o \) are the thickness of the inner and outer shells, respectively,

\( R_i, R_o \) are the mean radii of the inner and outer shells, respectively, and

\( A_T \) is the area of the sandwich cross section for one frame spacing \( (L_F) \).

A stress intensity \( \sigma_1 \) was then computed using the Hencky-Von Mises
The inelastic general-instability pressures $p_c$ for the titanium models were determined as follows: Values of $E_S$ and $E_t$ were determined as a function of $\sigma_i$ by using the stress-strain curves obtained from the uniaxial compression tests. Then $p_c$ was plotted as a function of $\sigma_i$ using Equation [1]. Similarly, the applied pressure $p$ was plotted against $\sigma_i$ using Equation [3]. The inelastic general-instability pressure for each model was then obtained from the intersection of the two curves of $p_c$ versus $\sigma_i$ and $p$ versus $\sigma_i$, as shown in Figure 21 for Model OV-4. The inelastic general-instability pressures thus obtained are listed in Table 2;
they compare very favorably with the respective experimental collapse pressures.

It is recalled that examination of Models OV-3 and OV-5 after collapse (see Figures 16 and 18) revealed extensive corrugation of the inner and outer shells between adjacent webs, indicative of gross yielding of the shell elements. Also, the observed collapse pressures were in excellent agreement with the computed values $p_c$; see Table 2. Since the inelastic general-instability pressures are also in good agreement with the experimental collapse pressures, it appears logical to conclude that possibly the short one-diameter model (OV-3) of the original design and the four-diameter model (OV-5) of the suggested redesign may be of such a balanced nature that in each case the two modes of axisymmetric plastic collapse and inelastic general instability occurred simultaneously.

The aforementioned procedure for determining the inelastic general-instability pressure $p_c$ for a web-stiffened sandwich cylinder may appear to be somewhat empirical. However, it does define the important feature of the dependence of the inelastic general-instability strength on the stress-strain characteristics of the material, and also affords a means for obtaining a good estimate of strength for this mode of collapse. Moreover, Models OV-3, OV-4, and OV-5 were machined from nascent bars of titanium alloy so that no appreciable residual stresses existed in these small-scale models, as would occur in a fabricated structure due to rolling and welding of flat plating. Such built-in stresses would effectively alter the shape of the stress-strain curve so that collapse strength of a fabricated sandwich hull may be considerably lower than that observed from tests of initially stress-free models. The question of how much strength reduction would occur can only be answered by testing some large-scale fabricated models in which the influences of residual stresses and out-of-roundness are present.

If the observed collapse pressure of 11,100 psi for Model OV-4 is extrapolated to a yield strength of 120,000 psi, a pressure of 9650 psi is obtained. This value is 3.5 percent below the minimum design collapse pressure of 10,000 psi. Since the mode of failure observed for Model OV-4 appears to be greatly influenced by the shape of the stress-strain curve of the material, the collapse pressure of the fabricated prototype
hull could be even lower. On the basis of these results it can be concluded
that the original design would prove inadequate in meeting the minimum
specified collapse pressure of 10,000 psi.

When the observed collapse pressure of Model OV-5, incorporating
the suggested redesign in a long cylinder length, is extrapolated to a yield
strength of 120,000 psi, a pressure of 11,290 psi is obtained. This is 13
percent above the minimum specified. The question as to whether this
represents an adequate margin for the fabricated hull remains unanswered
at the present. In addition, the question of fatigue strength for the sand-
wich hull may be of some concern since stresses on the order of 90 per-
cent of the minimum specified yield strength (120,000 psi) at an operating
depth of 15,000 ft are indicated by the analysis of Reference 3.

Before any final conclusions can be drawn concerning the adequacy
of the suggested redesign, it will be necessary to test larger scale tita-
nium models which are fabricated to be identical in all respects, except
size, to those anticipated in the prototype. This should be done so that all
factors which appear to influence static and fatigue strength are adequately
considered. At this stage of the preliminary design studies, it can only be
said that the OV-5 redesign may represent a possible hull structure for
the proposed oceanographic vehicle.

CONCLUSIONS

1. The test results obtained with the one-diameter-long model
(OV-3) indicate that the interbay strength of the original sandwich hull
design exceeds the desired collapse pressure of 10,000 psi. Also, the
construction technique of slipping the outer shell over the integral web-
and-inner-shell assembly, without physically joining the webs to the outer
shell, does not appear to prejudice structural strength. The test results
obtained with the four-diameter-long model (OV-4) indicate that the crit-
ical mode of failure is one of inelastic general instability and that the
original design is not adequate to satisfy the minimum specified collapse
pressure.
2. Tests of a machined model (OV-5), incorporating a redesign of the sandwich configuration, resulted in a collapse strength 13 percent higher than the minimum specified. The critical mode of failure for the redesign is the same as that for the original design. It is a mode which is greatly influenced by the shape of the stress-strain curve of the material in the structure.

3. Before any final conclusions can be drawn concerning the adequacy of the suggested redesign, it will be necessary to test larger scale fabricated models with built-in residual stresses and out-of-roundness to simulate the effects due to rolling and welding of plating as expected in the fabrication of the prototype.

ACKNOWLEDGMENTS

The authors are indebted to Mr. William E. Ball, Jr., who, prior to leaving the Model Basin, served as project manager of this program. He conducted the pertinent strength calculations and worked out the necessary details for the construction and instrumentation of Models OV-1 and OV-2. They are also indebted to Mr. John G. Pulos for his suggestions and advice.
REFERENCES


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HYDROSTATIC TESTS OF STRUCTURAL MODELS FOR
PRELIMINARY DESIGN OF A WEB-STIFFENED SANDWICH
PRESSURE HULL, by Kenneth Hom and William F. Blumenberg.

UNCLASSIFIED

A series of structural model tests was conducted to evaluate a
preliminary sandwich design of the pressure hull for a proposed
15,000-ft operating-depth oceanographic vehicle. The model stud-
ies were limited to an evaluation of the static strength of a typi-
cal section of the web-stiffened sandwich configuration. It is in-
tended that the prototype be fabricated of high-strength, α-titani-
um alloy having a nominal yield strength of approximately
120,000 psi.

1. Surveying ships--Design--Model tests
2. Surveying ships--Strength--Model tests
3. Titanium alloys--Strength--Evaluation
4. Pressure hulls--Sandwich construction--Model tests
I. Hom, Kenneth
II. Blumenberg, William F.
III. S-F013 03 02
IV. S-F013 01 03

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